[illegible]

[illegible]

[illegible]

[illegible]:

$$\begin{cases} u' = f(t, u), \ t \in [0, T] \\ u(0) = u_0 \end{cases}$$

[illegible] $[0, T]$ [illegible] M [illegible] $h = T/M$ [illegible]

[illegible] Euler [illegible] (FE)

$$u^{(n+1)} = u^{(n)} + hf\left(t^{(n)}, u^{(n)}\right).$$

[illegible] Euler [illegible] (BE)

$$u^{(n+1)} = u^{(n)} + hf\left(t^{(n+1)}, u^{(n+1)}\right).$$

[illegible] (LF)

$$u^{(n+1)} = u^{(n-1)} + 2hf\left(t^{(n)}, u^{(n)}\right).$$

[illegible] (Tz)

$$u^{(n+1)} = u^{(n)} + h\frac{f\left(f^{(n)}, u^{(n)}\right) + f\left(t^{(n+1)}, u^{(n+1)}\right)}{2}.$$

[illegible] (Accuracy) [illegible] (Stability) [illegible]

[illegible]

[illegible] (Local Truncation Error) $R^{(n)}$ [illegible][1] [illegible] $t^{(n)}$ [illegible] $t^{(n)}$ [illegible]

[illegible] *Euler* [illegible]

[illegible] Euler [illegible] $R^{(n)}$ [illegible]

$$R^{(n)} = \frac{u\left(t^{(n+1)}\right) - u\left(t^{(n)}\right)}{h} - u'\left(t^{(n)}\right).$$

[illegible] $u\left(t^{(n+1)}\right)$ [illegible] Taylor [illegible]

$$u\left(t^{(n+1)}\right) = u\left(t^{(n)}\right) + hu'\left(t^{(n)}\right) + \frac{h^2}{2}u''\left(\xi^{(n)}\right), \quad \xi^{(n)} \in \left[t^{(n)}, t^{(n+1)}\right].$$

[illegible]

$$R^{(n)} = \frac{h}{2}u''\left(\xi^{(n)}\right) = O(h), \quad \xi^{(n)} \in \left[t^{(n)}, t^{(n+1)}\right].$$

[1] [illegible] h [illegible]

[illegible] $E^{(n)}$ [illegible]

$$\frac{E^{(n+1)} - E^{(n)}}{h} + R^{(n)} = f\left(t^{(n)}, u\left(t^{(n)}\right)\right) - f\left(t^{(n)}, u^{(n)}\right).$$

[illegible] $R = \max_n \left|R^{(n)}\right|$[illegible] $e^{(n)}$ [illegible]

$$\begin{aligned}
\left|E^{(n+1)}\right| &\le \left|E^{(n)}\right| + hR + hL\left|u\left(t^{(n)}\right) - u^{(n)}\right| = (1 + hL)\left|E^{(n)}\right| + hR \\
&\le (1 + hL)^{n+1}\left|E^{(0)}\right| + \frac{R}{L}\left((1 + hL)^{n+1} - 1\right) \\
&\le e^{LT}\left|E^{(0)}\right| + \frac{R}{L}\left(e^{LT} - 1\right) \le O(h), n = 0, 1, \cdots, M - 1.
\end{aligned}$$

[illegible] *Euler* [illegible]

[illegible] Euler [illegible] $R^{(n)}$ [illegible]

$$R^{(n)} = \frac{u\left(t^{(n)}\right) - u\left(t^{(n-1)}\right)}{h} - u'\left(t^{(n)}\right).$$

[illegible] $u\left(t^{(n-1)}\right)$ [illegible] Taylor [illegible]

TODO: [illegible]

[illegible]

[illegible]

$$\begin{cases} u'(t) = f(t, u(t)) = -\cos(5\pi t)\tan(5\pi u), \\ u(0) = 1/60. \end{cases}$$

[illegible] $t \in [0, 1]$[illegible]

$$u(t) = \frac{1}{5\pi}\arcsin\left(\frac{\sqrt{6} - \sqrt{2}}{4\exp(\sin(5\pi t))}\right).$$

[illegible]

[illegible]

```
function [x, result] = forward_euler(T, step, u0, f)
    x = linspace(0, T, step + 1);
    h = T / step;
    result = zeros(1, step + 1);
    result(1) = u0;
    for k = 2:step + 1
        result(k) = result(k - 1) + h * f(x(k - 1), result(k - 1));
    end
end
```

[illegible]

```
function [x, result] = backward_euler(T, step, u0, f)
    x = linspace(0, T, step + 1);
    h = T / step;
    result = zeros(1, step + 1);
    result(1) = u0;
```

```
    for k = 2:step + 1
        temp_func = @(uk) result(k - 1) + h * f(x(k), uk) - uk;
        result(k) = fzero(temp_func, result(k - 1));
    end
end
```

[illegible]

```
function [x, result] = leap_frog(T, step, u0, u1, f)
    x = linspace(0, T, step + 1);
    h = T / step;
    result = zeros(1, step + 1);
    result(1) = u0;
    result(2) = u1;
    for k = 3:step + 1
        result(k) = result(k - 2) + 2 * h * f(x(k - 1), result(k - 1));
    end
end
```

[illegible]

```
function [x, result] = trapezoidal(T, step, u0, f)
    x = linspace(0, T, step + 1);
    h = T / step;
    result = zeros(1, step + 1);
    result(1) = u0;
    for k = 2:step + 1
        temp_func = @(uk) result(k - 1) - uk + ...
            h * (f(x(k), uk) + f(x(k - 1), result(k - 1))) / 2;
        result(k) = fzero(temp_func, result(k - 1));
    end
end
```